Quiz 2 Solution

Question 1

Since the 2 of clubs is an even numbered card, $C_2 \subset E$ so that $P[C_2E] = P[C_2] = 1/3$. Since $P[E] = 2/3$,

$$P[C_2|E] = \frac{P[C_2E]}{P[E]} = \frac{1/3}{2/3} = 1/2. \tag{1}$$

The probability that an even numbered card is picked given that the 2 is picked is

$$P[E|C_2] = \frac{P[C_2E]}{P[C_2]} = \frac{1/3}{1/3} = 1. \tag{2}$$

Question 2

1) 52! = 52 x 51 x 50 x …. x 2 x 1.
2) $\binom{11}{5}$ = (11!/(5! 6!))
3) 2^{10}

Question 3

Decoding each transmitted bit is an independent trial where we call a bit error a "success." Each bit is in error, that is, the trial is a success, with probability p. Now we can interpret each experiment in the generic context of independent trials.

(1) The random variable X is the number of trials up to and including the first success. Similar to Example 2.11, X has the geometric PMF

$$P_X(x) = \begin{cases} p(1-p)^{x-1} & x = 1, 2, \ldots \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

(2) If $p = 0.1$, then the probability exactly 10 bits are sent is

$$P[X = 10] = P_X(10) = (0.1)(0.9)^9 = 0.0387 \tag{2}$$

The probability that at least 10 bits are sent is $P[X \geq 10] = \sum_{x=10}^{\infty} P_X(x)$. This sum is not too hard to calculate. However, its even easier to observe that $X \geq 10$ if the first 10 bits are transmitted correctly. That is,

$$P[X \geq 10] = P[\text{first 10 bits are correct}] = (1-p)^{10} \tag{3}$$

For $p = 0.1$, $P[X \geq 10] = 0.9^{10} = 0.3487$.

(3) The random variable Y is the number of successes in 100 independent trials. Just as in Example 2.13, Y has the binomial PMF

$$P_Y(y) = \binom{100}{y} p^y (1-p)^{100-y} \tag{4}$$

If $p = 0.01$, the probability of exactly 2 errors is

$$P[Y = 2] = P_Y(2) = \binom{100}{2} (0.01)^2 (0.99)^{98} = 0.1849 \tag{5}$$

(4) The probability of no more than 2 errors is

$$P[Y \leq 2] = P_Y(0) + P_Y(1) + P_Y(2) \tag{6}$$

$$= (0.99)^{100} + 100(0.01)(0.99)^{99} + \binom{100}{2}(0.01)^2(0.99)^{98} \tag{7}$$

$$= 0.9207 \tag{8}$$

(5) Random variable Z is the number of trials up to and including the third success. Thus Z has the Pascal PMF (see Example 2.15)

$$P_Z(z) = \binom{z-1}{2} p^3 (1-p)^{z-3} \tag{9}$$

Note that $P_Z(z) > 0$ for $z = 3, 4, 5, \ldots$.

(6) If $p = 0.25$, the probability that the third error occurs on bit 12 is

$$P_Z(12) = \binom{11}{2} (0.25)^3 (0.75)^9 = 0.0645 \tag{10}$$